Junior Division: Problems 2 : Solutions

J1. Six straight lines have been drawn on a plane so that they are all distinct, none of them are parallel, and no three intersect at the same point. Into how many regions has this plane been subdivided?

Solution

The first line divides the plane into 2.

The second line crosses the existing 1 line, making 2 further regions. 2 + 2 = 4 regions in all.

When a line is added it never goes through an existing crossing point since no 3 lines intersect at a point. So always the stated number of further regions are added.

The next line crosses the existing 2 lines, making 3 further regions. 4 + 3 = 7 regions in all.

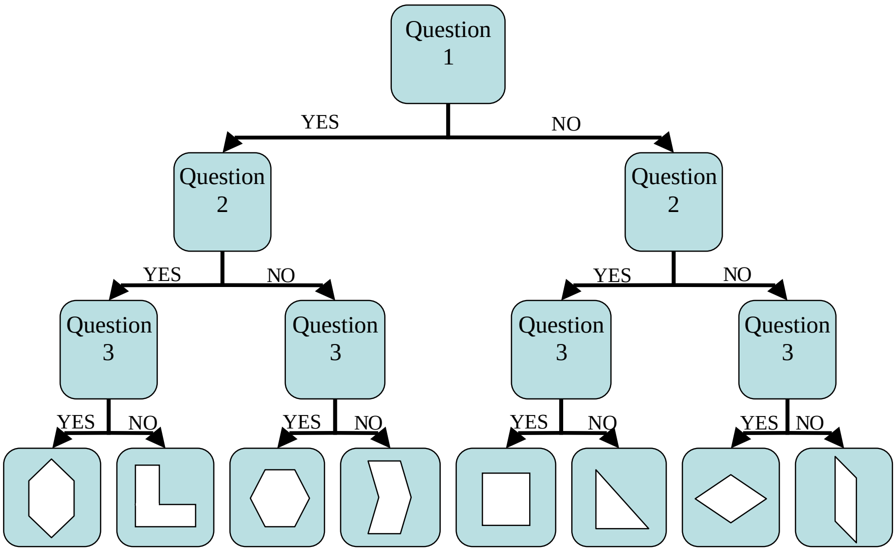
The next line crosses the existing 3 lines, making 4 further regions. 7 + 4 = 11 regions in all.

The next line crosses the existing 4 lines, making 5 further regions. 11 + 5 = 16 regions in all.

The next line crosses the existing 5 lines, making 6 further regions. 16 + 6 = 22 regions in all.

So the plane is divided into 22 regions.

J2.



The eight geometrical shapes shown in the boxes in the bottom row of the flowchart have been sorted by answering questions 1, 2 and 3 in turn. Your task is to find three questions to sort the shapes in the way shown.

For example, if Question 1 was “Are all the sides the same length?” the first shape would correctly follow the YES branch but the square would also follow the YES branch, which is not correct.

Solution

Question 1:	Is it a hexagon ?	or	Does it have 6 sides?
Question 2:	Does it have a right angle ?		
Question 3:	Is there a vertical line of symmetry?	or	Is there more than one line of symmetry?

J3. In a particular exam, the ratio of the number of pupils who passed to the number of pupils who failed was 3:2.

If the pass mark had been lowered so that 12 more pupils passed then the ratio of passes to fails would have been 21:10.

How many pupils passed the exam?

Solution

Originally,	no. of passes = x	no. of fails = $\frac{2}{3}x$
If the pass mark changes then	no. of passes = $x + 12$	no. of fails = $\frac{2}{3}x - 12$

The latter is in the ratio 21:10 so

$$\frac{2}{3}x - 12 = \frac{10}{21}(x + 12)$$

giving

$$x = 93$$

So, 93 pupils passed the exam.

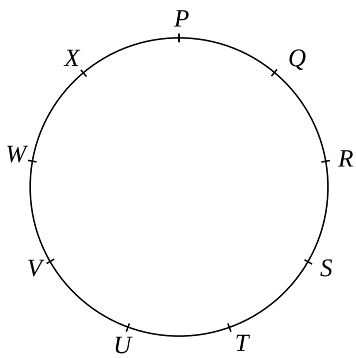
J4. A man is walking his dog on the lead towards home at a constant speed of 3 m.p.h. When they are 1.5 miles from home, the man lets the dog off the lead. The dog immediately runs towards home at a constant speed of 5 m.p.h. When the dog reaches the house, it turns round and runs back to the man at the same speed. When it reaches the man, it turns back for the house. This is repeated until the man reaches the house and lets the dog in. How many miles does the dog run from being let off the lead to being let into the house?

Solution

The man covers 1.5 miles at 3 m.p.h and so walks for ½ hour.

During this time the dog is running at 5m.p.h. and so covers 2.5 miles.

J5. The nine points P, Q, R, S, T, U, V, W, X lie equally spaced round the circumference of a circle, as shown in the diagram. Find the number of distinct triangles whose vertices belong to the set $\{P, Q, R, S, T, U, V, W, X\}$ so that the centre of the circle lies in the interior of each triangle.



Solution

The "length" of a side of a triangle will be given by the distance apart of the end points round the circle. We count the number of triangles by the length of the shortest side.

If the length of the shortest side is 1, e.g. *PQ*, then there is only one triangle e.g. *PQU* with the centre of the circle in the interior. Thus there are in total 9 triangles with shortest side of length 1. Note that the other two sides have length 4.

If the length of the shortest side is 2, e.g. *PR* then there are two triangles e.g. *PRU*, *PRV* with the centre in the interior. There are thus 18 triangles whose shortest side has length 2. Note that the other two sides have lengths 3 and 4.

If the length of the shortest side is 3, then all sides must have length 3 and there are only 3 distinct such triangles.

Note that if a triangle has a side of length 4 then some other side must have length less than 3. Hence a side of length 4 (or more) cannot be the shortest side of any triangle.

So in total there are $9 + 18 + 3 = 30$ such triangles.