Junior Division: Problems 2 − solutions

J1. Four nearby primary schools each have a basketball team – the Plodders, Ramblers, Galumphers and Wanderers. Their colours are purple, red, green and white and the captains are Parker, Richards, Grainger and Watson.

In no case is the first letter of either colour or captain's name the same as that of the team. Similarly, in no case is the first letter of a captain's name the same as that of their colour.

The Wanderers play in green.

Grainger is the captain of the Plodders.

The first letter of the colour in which Watson plays is the first letter of the name of the captain who plays in red.

Who is the captain of the Wanderers?

Solution

Consider what is possible:

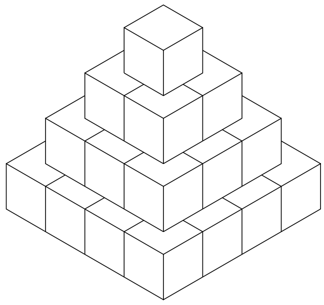
	Colour	Captain
Plodders	red white	Grainger
Ramblers	purple white	Watson Parker
Galumphers	red purple white	Parker or Watson Richards or Watson Parker or Richards
Wanderers	green	Parker or Richards

If Watson plays in red then the man who plays in red must be Richards but Watson is playing in red so this option does not work out. So Watson must play in purple and the man who plays in red must be Parker

	Colour	Captain
Plodders	red **white**	**Grainger**
Ramblers	**purple** white	**Watson** Parker
Galumphers	**red** purple white	**Parker** or Watson Richards or Watson Parker or Richards
Wanderers	**green**	Parker or **Richards**

This leaves Richards as the captain on the Wanderers.

J2. The diagram shows a pyramid made up of 25 cubes, each measuring 1 cm by 1 cm by 1 cm. The centre of the pyramid is a pyramid-shaped hollow, with each layer having edges one cube thick. What is the total surface area of the whole pyramid (including its hollow base)?



Solution

View the pyramid from above. The area of the upward facing faces is 16 square cm.

View the pyramid from below. The area of the downward facing faces is 16 square cm.

The bottom layer has outward facing faces with area $4 \times 4 = 16$ square cm.

The next layer has outward facing faces with area $4 \times 3 = 12$ square cm.

The next layer has outward facing faces with area $4 \times 2 = 8$ square cm.

The top layer has outward facing faces with area $4 \times 1 = 4$ square cm.

The bottom layer has inward facing faces with area $4 \times 2 = 8$ square cm.

The next layer has inward facing faces with area $4 \times 1 = 4$ square cm.

So the total surface area of the whole pyramid is 84 square cm.

J3. A number of aliens, who have just dropped in from another planet, emerge from their spacecraft.

(1) There is more than one alien.

(2) Each alien has more than one finger.

(3) Each alien has the same number of fingers.

(4) The total number of fingers on all the aliens is between 200 and 300.

(5) If you knew the total number of fingers of all the aliens you would know the number of aliens.

How many aliens were there?

Solution

If the total number of fingers has two different factors greater than one, say a and b, then you could have a aliens with b fingers each or b aliens with a fingers each and this would violate (5).

If the total number of fingers was a prime number then there would either be just one alien, violating (1) or each alien would have just 1 finger, violating (2).

So the total number of fingers must be the square of a prime number.

The only prime whose square is between 200 and 300 is 17 (since $17^2 = 289$). Thus there were 17 aliens.

J4. Three expert logicians played a game with a set of 11 cards each with a different two-digit prime number below 50. Each drew a card and held it up so that they could only see the number on their own card but could not see the other numbers. Ali, Bobby and Charlie in turn were then asked two questions, namely "Is your number the smallest of the three?" and "Is your number the largest of the three?". In the first round all three answered "Don't know" to both questions. The same happened in rounds two and three. In round 4 Ali answered "No" to the first question.

What numbers did each logician have?

Solution

{*This question describes an impossible situation! Many apologies - the setter is clearly not an expert logician because they did not take account of the possible "No" answers. Many thanks to the family who emailed us with an explanation. The mark guidelines take account of the error.*}

There are exactly 11 two-digit primes:

11	13	17	19	23	29	31	37	41	43	47

It is only the order which is important, so consider the numbers 1, 2, ... 11 instead.

Round 1

If A could see the 1 card, she would know she had the smallest number and would answer yes to the first question.

If A could see the 11 card or the 10 card she would know she definitely didn't have the smallest card and would answer no to the first question.

If A could see the 11 card, she would know she had the largest number and would answer yes to the second question.

If A could see the 1 card or the 2 card she would know she definitely didn't have the largest card and would answer no to the second question.

So after two "Don't know" answers everyone knows that A cannot have 1, 2, 10 or 11, and so must hold 3, 4, 5, 6, 7, 8 or 9.

Similarly, after the next two "Don't know" answers, everyone knows that B can only hold 3, 4, 5, 6, 7, 8 or 9.

If C could see the 1 card or the 2 card or the 3 card, she would know she had the smallest number and would answer yes to the first question.

If C could see the 11 card or the 10 card or the 9 card or the 8 card she would know she definitely didn't have the smallest card and would answer no to the first question.

If C could see the 11 card or the 10 card or the 9 card, she would know she had the largest number and would answer yes to the second question

If C could see the 1 card or the 2 card or the 3 card or the 4 card she would know she definitely didn't have the largest card and would answer no to the second question.

After the next two "Don't know" answers, everyone knows that C can only hold 5, 6 or 7.

Round 2

Like with C in the first round, after the next two "Don't know" answers, everyone knows that A can only hold 5, 6 or 7.

If B held 3, 4 or 5 he must have the smallest card and would answer yes to the first question.

If B held 9, 8, 7 or 6 he would know he definitely didn't have the smallest card and would answer no to the first question.

So it is not possible for B to answer "Don't know" to his first question in round 2 (or his second question either).

J5. Alistair and Jonny cross a lake by swimming and using a one-seat canoe. Each swims at 2 km/hour and paddles the canoe at 7 km/hour. They set off from the same point at the same time, heading straight for the boathouse at the opposite side, with Alistair swimming and Jonny paddling the canoe. After a while Jonny stops paddling, gets out of the canoe and immediately starts swimming. When Alistair reaches the canoe, which has not moved since Jonny started swimming, Alistair climbs in and immediately starts paddling. After 90 minutes they both arrive at the boathouse together. For how long was the canoe stationary?

Solution 1

Let t_1 hours be the length of time during which Alistair paddles and Jonny swims.

Let t_2 hours be the length of time during which Alistair swims and Jonny swims; the canoe is not moving during this time.

Let t_3 hours be the length of time during which Alistair swims and Jonny paddles.

Let d km be the total distance across the lake.

Since Alistair paddles at 7 km/h and swims at 2 km/h, then

$$7t_1 + 2t_2 + 2t_3 = d. \tag{1}$$

Since Jonny paddles at 7 km/h and swims at 2 km/h, then

$$2t_1 + 2t_2 + 7t_3 = d. \tag{2}$$

Since the canoe travels at 7 km/h and does not move while both Alistair and Jonny are swimming, then

$$7t_1 + 0t_2 + 7t_3 = d. \tag{3}$$

Since Alistair and Jonny each take 90 minutes ($1\frac{1}{2}$ hours) to cross the lake, then the total time gives

$$t_1 + t_2 + t_3 = \tfrac{3}{2}. \tag{4}$$

From (1) and (2) we obtain

$$7t_1 + 2t_2 + 2t_3 = 2t_1 + 2t_2 + 7t_3 \text{ or } 5t_1 = 5t_3.$$

So $$t_1 = t_3.$$

Substituting into (1) and (3) gives

$$7t_1 + 2t_2 + 2t_3 = 7t_1 + 0t_2 + 7t_3 \text{ so } 2t_2 = 5t_3. \text{ Hence } t_2 = \tfrac{5}{2}t_3.$$

From (4) $t_1 + \frac{5}{2}t_1 + t_1 = \frac{3}{2}$ and so $t_1 = \frac{1}{3}$.

Then $t_2 = \frac{5}{2}t_1 = \frac{5}{2}\left(\frac{1}{3}\right) = \frac{5}{6}$.

Hence the canoe is not moving for $\frac{5}{6}$ hours = 50 minutes.

Solution 2

Each must swim half the distance and paddle half the distance − otherwise the one who paddled further would arrive first.

Let the distance to the boathouse be d km. Then since time = distance/speed

time taken in hours = $\left(\frac{1}{2}d\right)\frac{1}{2} + \left(\frac{1}{2}d\right)\frac{1}{7} = \frac{d(7+2)}{28} = \frac{3}{2}$ so $d = \frac{14}{3}$.

Time to paddle 14/3 km at 7 km/h is $\left(\frac{14}{3}\right)\left(\frac{1}{7}\right)$ h = $\frac{2}{3}$ h = 40 mins

So canoe is stationary for $90 - 40 = 50$ minutes.