Junior Division: Problems 1

J1. In the diagram (not to scale) the rectangle is divided into nine smaller rectangles. The areas of five of the smaller rectangles are given. Determine the area of the rectangle labelled R.

5		2
	4	R
3	2	

Solution

In column 2 the area of the middle rectangle is twice the area of the bottom rectangle, so similarly in column 1 the area of the middle rectangle is $2 \times 3 = 6$.

In row 1 the right-hand rectangle has 2/5 the area of the left-hand rectangle, so similarly in the middle row the area R is $\dfrac{2}{5} \times 6 = 2.4$.

5		2
6	4	$R=2.4$
3	2	

So the area of the rectangle labelled R is 2.4.

J2. Forest Rovers, Southside United, Hilltown Thistle and Valley Wanderers were to play each other at football. After some of the matches had been played, a table showing some details of matches played, won, lost, drawn etc looked like this:

	Played	Won	Lost	Drawn	Goals for	Goals against	Points
Forest Rovers	2			1	0		
Southside United							0
Hilltown Thistle	1						
Valley Wanderers			0	0	4	2	6

3 points are given for a win and 1 for a draw. Complete the table and find the score in each match played, explaining how you worked it out.

Solution

Southside United has 0 points and so did not win or draw a match. Valley Wanderers did not draw a match so Forest Rovers and Hilltown Thistle must have drawn 0-0 since Forest Rovers scored no goals.

Hilltown Thistle only played one match, drawn 0-0, so they won 0, lost 0, drew 1 and had 0 goals for, 0 goals against and 0 points overall.

Valley Wanderers have 6 points and so must have won 2 matches against Forest Rovers and Southside United. Also, they had 2 goals scored against them so they must have won $3 - 2$ in one game and $1 - 0$ in the other. So Valley Wanderers played 2 and won 2.

Forest Rovers scored no goals for, so their match with Valley Wanderers was lost 0-1. So Forest Rovers won 0, lost 1 and had 1 goal against and 1 point overall.

Southside United lost 2-3 to Valley Wanderers and played no other match. So they played 1, won 0, lost 1, drew 0 and had 2 goals for, 3 goals against and 0 points overall.

The completed table is

	Played	Won	Lost	Drawn	Goals for	Goals against	Points
Forest Rovers	2	0	1	1	0	1	1
Southside United	1	0	1	0	2	3	0
Hilltown Thistle	1	0	0	1	0	0	1
Valley Wanderers	2	2	0	0	4	2	6

Scores in the three matches played were

      Forest Rovers 0, Hilltown Thistle 0
   Forest Rovers 0, Valley Wanderers 1
      Southside United 2, Valley Wanderers 3

J3. *A* and *B* are two towns connected by a single road which crosses a bridge over a wide river. James leaves *A* at 10.38 am and walks along the road to *B* at uniform speed, reaching *B* at 1.50 pm. On the same day Isla leaves *B* at 9.20 am and walks along the road to *A* at uniform speed, reaching *A* at 12 noon. They arrive at their nearest end of the bridge at the same time. James leaves the bridge one minute later than Isla.

At what time did they reach the bridge?

Solution

James takes 192 minutes to walk between *A* and *B* and Isla takes 160 minutes. So James takes 1.2 minutes to walk the distance Isla covers in 1 minute. So James takes 6 minutes to cross the bridge and Isla takes 5 minutes.

bridge

10.38 t 6   1.50   James 192 min

A   B

12.00 $5t/6$ 5 $t + 78$ 9.20   Isla 160 min
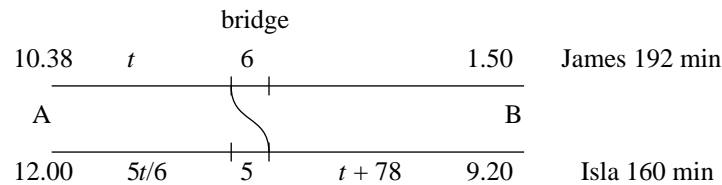
Let James take t minutes to reach the bridge.

Isla takes $t + 78$ minutes to reach the bridge, and a further 5 minutes to reach the far end of the bridge. But Isla walks faster than James, and so will only need $\frac{5}{6}t$ minutes to complete her journey. Hence

$$t + 78 + 5 + \frac{5}{6}t = 160$$

$$\frac{11}{6}t = 77$$

$$t = 42$$

Thus they reach the bridge 42 minutes after 10.38 am which is 11.20 am.

J4. At present, the sum of the ages of the parents, P, is five times the sum of the ages of their children, C. Two years ago, the sum of the ages of the husband and wife was eleven times the sum of the ages of the same children. A year from now, it will be four times the sum of the ages of the same children.

Determine the number of children.

Solution

Let the number of children be n.

Now
$$P = 5C.$$

Two years ago
$$P - 4 = 11(C - 2n).$$

A year from now
$$P + 2 = 4(C + n).$$

So
$$5C - 4 = 11C - 22n$$
$$22n - 4 = 6C$$
$$3C = 11n - 2$$

and
$$5C + 2 = 4C + 4n$$
$$C = 4n - 2.$$

Hence
$$0 = n - 4$$
$$n = 4.$$

There are 4 children.

J5. Two numbers x and y satisfy three of the following equations but do not satisfy the remaining one.

$$x + y = 63 \qquad x - y = 47 \qquad xy = 392 \qquad \frac{x}{y} = 8$$

What is the value of x?

Solution

Solving the first two equations gives $x = 55$ and $y = 8$.

But this does not satisfy either of the other two equations, so the equation not satisfied must be one of the first two.

Hence the other two equations are satisfied. Solving these gives

$$xy\frac{x}{y} = 392 \times 8$$

$$x^2 = 49 \times 8 \times 8.$$

So $x = 56$ and $y = 7$.

Since $56 + 7 = 63$ and $56 - 7 = 49$, these values of x and y satisfy the first equation but not the second.

So the value of x is 56.