Primary Division: Problems II Solutions

P2.1. On a ski lift the chairs are equally spaced. They are numbered in order from 1. Kieran went skiing. He got in chair 10 to go to the top of the slopes. Exactly half way to the top, he passed chair 80 on its way down.
How many chairs were there on the ski lift?
Explain your reasoning.

Solution

Chairs 11 to 79 are between between chair 10 and chair 80, so there are 69 chairs in between.

So there are also 69 chairs in the other half of the circuit. 9 of them are chairs 1 to 9, so the other 60 are chairs 81 to 140.

Hence there are 140 chairs in all.

P2.2. Katy writes down all of the integers from 1 to 1000 that have 4 as the sum of their digits.
   (a) List all such numbers.
   (b) Find the proportion of these numbers that are not prime.

Solution

(a) The numbers are

$$4, 40, 400$$
$$\mathbf{13}, \mathbf{103}, 130, \mathbf{31}, 301, 310$$
$$22, 202, 220$$
$$112, 121, \mathbf{211}$$

(b) Only the 4 numbers in bold are prime out of the total of 15 numbers.

So the proportion of these numbers that are not prime is $\dfrac{11}{15}$.

P2.3. Lunnocks sell chocolate biscuits in packs of six, whereas Tees sell a similar type of biscuit, but only in packs of five. At a conference of mathematics teachers the organisers wish to provide exactly one biscuit per person at the coffee break. Can this be done for 58 people?
What is the largest number of people attending where it is not possible to purchase the exact number of biscuits required?
Explain your reasoning.

Solution 1

Some very small events could not be catered for, but it is clear that 50 can be done.
The table shows one way if doing it:

Conference size	50	51	52	53	54	55	56	57	58
Packs of Lunnocks (6)	0	1	2	3	4	5	6	7	8
Packs of Tees (5)	10	9	8	7	6	5	4	3	2

So it is possible to cater for a conference of 58.

Looking at small numbers, this table shows which events **can** be catered for.

Conference size	5	6	…	10	11	12	…	15	16	17	18	…	20
Packs of Lunnocks (6)	0	1		0	1	2		0	1	2	3		0
Packs of Tees (5)	1	0		2	1	1		3	2	1	0		4

The gaps between 'catered conferences' decrease in size and vanish beyond 20. So the largest conference which cannot be catered for is 19.