Primary Division: Solutions to Problems III

P3.1. One of the children bought their teacher a mathematical plant for the classroom windowsill.
At the end of the first day it had increased its height by a factor of a half.
At the end of the second day it had increased its height from the end of day 1 by a factor of a third.
The class found that this pattern continued day after day.
How many days did it take to grow to 10 times its original height?

Solution

Suppose the initial height is h cm then,

End of day 1: height is $\frac{3h}{2} = 1\tfrac{1}{2}h$

End of day 2: height is $\frac{3h}{2} \times \frac{4}{3} = \frac{4h}{2} = 2h$

End of day 3: height is $\frac{4h}{2} \times \frac{5}{4} = \frac{5h}{2} = 2\tfrac{1}{2}h$

End of day 4: height is $\frac{5h}{2} \times \frac{6}{5} = \frac{6h}{2} = 3h$

End of day 5: height is $\frac{6h}{2} \times \frac{7}{6} = \frac{7h}{2} = 3\tfrac{1}{2}h$

$\vdots$ $\quad\quad$ $\ldots$

End of day n: height is $\frac{(n+2)h}{2}$

10 times its original height would be $10h$ so,

$$\frac{(n+2)h}{2} = 10h$$

$$n + 2 = 20$$

$$= 18$$

It took 18 days.

P3.2. Three buckets are coloured red, green and blue. Each bucket contains four balls numbered 1, 2, 3, and 4, of the same colour as the bucket. Without looking, Emily chooses one ball at random from each of the buckets. If r, g and b are the numbers on the balls chosen from the red, green and blue buckets respectively, Emily wins a prize when $r = g + b$.
What is the probability that Emily wins a prize?

Solution
There are $4 \times 4 \times 4 = 64$ possible different, equally likely choices of the three balls.

Think about the green and blue choices that could win:

g	b
1	1
1	2
2	1
2	2
3	1
1	3

All other choices have a total of more than 4 and so lose.

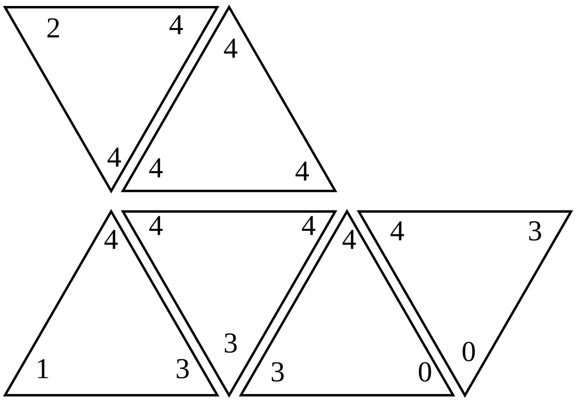
In each case, only one of the 4 red balls will give the required total.

So there are 6 ways to win out of the 64 possible choices.

Emily wins a prize with probability $\frac{6}{64} = \frac{3}{32}$.

P3.3. Three-sided dominoes are equilateral triangles and one face of each domino has a number in each corner and the other side is blank. The numbers range from 0 up to the highest number in the set, 4. Here is an example of a game which started with the 4 4 4 domino.

The set contains all possible different dominoes. How many dominoes are there in the set?



Solution

List the dominoes systematically, starting with the one with 0 in each corner, then adding those with at least one 1, then those with at least one 2 and so on. Dominoes with three different numbers have a mirror image − see the 4 3 0 domino in the example game.

0 0 0 i.e. 1 way of creating dominoes with numbers up to 0.

1 0 0
1 1 0
1 1 1 i.e. 3 ways of creating dominoes with a 1 and other numbers not greater than 1.

2 0 0
2 1 0
2 0 1 (a mirror image of the previous one)
2 1 1
2 2 0
2 2 1
2 2 2 i.e. 7 ways of creating dominoes with a 2 and other numbers not greater than 2.

3 1 0
3 0 1
3 2 0
3 0 2
3 2 1
3 1 2 (3 ways of choosing the first different number after a 3, and 2 ways of choosing the last different number i.e. 6 dominoes in all.)

3 0 0
3 1 1
3 2 2 (3 ways of choosing a repeated number after a 3)

3 3 0
3 3 1
3 3 2 (3 ways of choosing a number after a repeated 3)

3 3 3 (1 way)
i.e. 6 + 3 + 3 + 1 = 13 ways of creating dominoes with a 3 and other numbers not greater than 3.

4 ways of choosing the first different number after a 4, and 3 ways of choosing the last different number i.e. $4 \times 3 = 12$ dominoes in all.

4 ways of choosing a repeated number after a 4

4 ways of choosing a number after a repeated 4

1 way of choosing 4 4 4

i.e. $12 + 4 + 4 + 1 = 21$

So there are 1 + 3 + 7 + 13 + 21 = 45 ways of creating dominoes with a 4.

There are 45 3-sided dominoes in the set.