2019−2020 Primary Division Problems II Solutions

P2.1. In a recent by-election the original numbers were lost but the live news broadcast at the time reported:

> "Jones and Brown together polled 65% of the total number of votes; Jones and Robinson together accounted for 15355 votes and Brown polled 8245."

There were no other candidates.
Which of the three candidates had the most votes? **Explain why.**

Solution:
We know

$$\begin{aligned} J + B &= 65\% \text{ of } (J + B + R) \\ \text{and} \quad J + R &= 15\,355 \\ \text{and} \quad B &= 8245. \\ \text{So the total votes} &= (J + R) + B = 15\,355 + 8245 \\ &= 23\,600. \\ \text{So} \quad J + B &= 65\% \text{ of } 23\,600 \\ &= 15\,340. \\ \text{So} \quad J &= 15\,340 - 8245 = 7095 \\ \text{and} \quad R &= 15\,355 - 7095 = 8260. \end{aligned}$$

Answer: Robinson got the most votes.

P2.2. Eighteen years ago Ivan was three times as old as his son but he is only twice as old now. How old is Ivan?

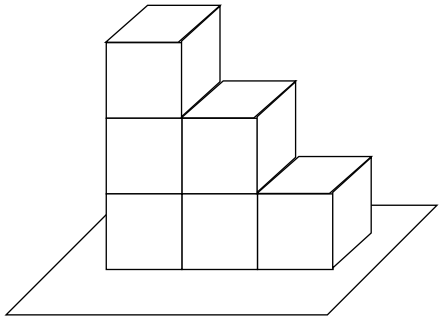
Solution
Let Ivan's current age be x years. So, his son is $\frac{1}{2}x$.

Eighteen years ago, Ivan was $(x - 18)$ and the son would have been $\left(\frac{1}{2}x - 18\right)$.
So,

$$\begin{aligned} x - 18 &= 3\left(\frac{1}{2}x - 18\right) = \frac{3}{2}x - 54 \\ 2x - 36 &= 3x - 108 \\ x &= 108 - 36 \\ &= 72 \end{aligned}$$

Ivan is 72 years old.

P2.3. Six dice are placed on a table as shown.

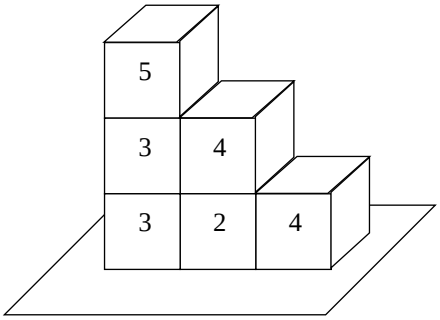


Each die is of the standard type with 1 opposite 6, 2 opposite 5 and 3 opposite 4.

What is the minimum sum of all the 21 visible faces of the dice in the stack?
(The *invisible* faces are those between the dice and the faces on the table.)

Solution

In the diagram alongside the number on each die shows **how many** of its faces are visible.



To keep the total of the exposed faces as low as possible, the hidden faces must be the largest possible.

5 faces exposed, hide the 6 face. {Sum of exposed faces is 1+2+3+4+5 = 15 for this die.}
4 faces exposed, hide the 6 and 5 faces. (This is possible since the hidden faces are adjacent not opposite.) {Sum of exposed faces is 1+2+3+4 = 10 for these 2 dice.}
When 3 faces are exposed, 2 of the faces are opposite, totalling 7. Make the third exposed face a 1, giving an overall sum of 8 for these 2 dice.
When 2 faces are exposed, the 2 faces are opposite, totalling 7 for this die.
So the overall minimum sum for the six dice is $15 + 2 \times 10 + 2 \times 8 + 7 = 58$.

Or, equivalently

Hidden faces	1	2	3	4	
Amounts hidden	6	$5 + 6$	$7 + 6$	2×7	
	$= 6$	$= 11$	$= 13$	$= 14$	
Multiply by	1	2	2	1	
gives	6	22	26	14	$= 68$

The total values of all the faces is $(1 + 2 + 3 + 4 + 5 + 6) \times 6 = 126$.
So the visible faces total $126 - 68 = 58$.