Primary Division 2018-2019 Round 3 Solutions

P3.1. One of the stories in circulation at the moment is a cautionary tale. A few years ago, a local man fearing an imminent financial crisis left his job in the bank and set up an eco-fruit farm. He began well but knew little of either crop rotation or the science of fertilising. The result was that each year his crop was 25% less than in the previous year. The fall between 2007 and 2008 was 64 kg. What was his crop in 2011? **Explain**.

Solution 1

As 64 kg was 25% so in 2007 the weight of the crop was $4 \times 64 = 256$ kg;

the crop in 2008 was $256 - \frac{1}{4} \times 256 = 192$;

the crop in 2009 was $192 - \frac{1}{4} \times 192 = 192 - 48 = 144$;

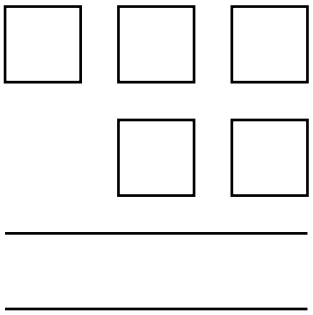
the crop in 2010 was $144 - \frac{1}{4} \times 144 = 144 - 36 = 108$;

the crop in 2011 was $108 - \frac{1}{4} \times 108 = 108 - 27 = 81$ kg.

Solution 2

64 kg was 25% so in 2007 the weight of the crop was $4 \times 64 = 256$ kg;
The crop in each year was $\frac{3}{4}$ of the crop in the previous year.
So in 2011, after 4 years, the crop was $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times 256 = 81$kg.

P3.2. Each of the digits 2, 3, 5, 7 and 8 is placed one to a box in the diagram.



(a) If the two-digit number is subtracted from the three digit number, what is the smallest possible difference?

(b) If the three-digit number is multiplied by the two-digit number, what is the smallest possible product?

Solution

(a) To make the difference small, we need the *smallest* possible 3-digit number and the *largest* possible 2-digit number.

In the 3-digit number, choose the hundreds first, then the tens and finally the units. So the smallest number of hundreds is 2, the smallest number of tens is 3 and the smallest number of units is 5. The number is 235.

In the 2 digit number, choose the tens first and then the units. So the largest number of tens is 8 and we are left with 7 units. The number is 87.

The smallest difference is then $235 - 87 = 148$.

(b)(i) We have to investigate a product $\square\square\square \times \square\square$. We have 5 digits to fill in. So there are 5 ways of filling the first digit, 4 ways of filling the second so $5 \times 4 = 20$ ways of filling the first two which extends to $5 \times 4 \times 3 \times 2 \times 1 = 120$ for all 5 digits. So checking all these is not a good way!

We want a small product so using small values in the leading places looks a good start. So consider $2 \times 100 \times 3 \times 10$. This is 6000. If we replace 3 by 5, we get $2 \times 100 \times 5 \times 10 = 10000$ which is bigger not smaller. (Replacing the 2 by 5 gives $5 \times 100 \times 3 \times 10 = 15000$.) If we get an answer which exceeds 10000 we would need to review these but for now let us consider the leading digits to be 2 and 3.

We can always reduce a product by ensuring that the digits are increasing within each number e.g. 375×82 can be reordered to 357×28 to get a smaller product.

So we need to try all possibilities for $2\cdot \times 3\cdot\cdot$ and $3\cdot \times 2\cdot\cdot$.

$2\cdot \times 3\cdot\cdot$	$25 \times 378 = 9450$	$27 \times 358 = 9666$	$28 \times 357 = 9996$
$3\cdot \times 2\cdot\cdot$	$35 \times 278 = 9730$	$37 \times 258 = 9546$	$38 \times 257 = 9766$

The smallest of these is $25 \times 378 = 9450$ which is less than 10000.

So the smallest possible product is 9450.

{A much more complicated solution as included in the solution of J1.}

P3.3. A jeweller makes sets of small cubes out of solid silver. The jeweller has gold-plated none, some, or all of the faces on some of the cubes. The cubes in a set are all different, and no other cube can be added to the set. How many cubes are there in a set?

Solution 1

Each cube has 6 faces so we need to identify the number of different cubes with each of 0, 1, 2, 3, 4, 5 or 6 faces gold-plated

Number of faces gold-plated	Number of different cubes
0	1
1	1
2	1 with an edge in common
	1 which opposite faces plated
3	1 where all 3 faces have a common corner
	1 where 3 faces are U-shaped
4 (the opposite of 2 faces)	1 and 1
5 (the opposite of 1 face)	1
6 (the opposite of 0 faces)	1

So there are 10 cubes in the set.

Solution 2

gold faces	
0	
1	
2 touching	Place the cube on a table on a gold face. The second gold face can be on top (opposite the first gold face) or on a vertical side (touching the first gold face).
2 opposite	
3 corner	Place the cube on a table on a gold face. The the second gold face can be on top, and so the third gold face must complete a row of 3 gold faces. Alternatively it can be on a vertical side. Then the third gold face may be an adjacent vertical side (the 3 faces touch at a corner). Or the third gold face can be an opposite vertical side or the top face: either way the three gold faces are in a row as before.
3 row	
4	(silver touching)
4	(silver opposite)
5	
6	

So there are 10 cubes in the set.