Primary Division 2017-2018 Round 2 Solutions

P2.1. A few days after his birthday, a retired mathematics teacher was musing. He realised that his age was a prime number. He then noticed that a year ago, his age was the product of 3 distinct prime numbers and, a year hence his age would be the product of a square and a cube.
How old is he?

Solution 1
First we list the prime numbers above 40 and less than 100:

$$41, \ 43, \ 47, \ 53, \ 59, \ 61, \ 67, \ 71, \ 73, \ 79, \ 83, \ 89, \ 97$$

Now consider the numbers which are 1 less than each of these and how they factorise:

$40 = 2 \times 2 \times 2 \times 5$	$42 = 2 \times 3 \times 7$	$46 = 2 \times 23$
$52 = 2 \times 2 \times 13$	$58 = 2 \times 29$	$60 = 2 \times 2 \times 3 \times 5$
$66 = 2 \times 3 \times 11$	$70 = 2 \times 5 \times 7$	$72 = 2 \times 2 \times 2 \times 3 \times 3$
$78 = 2 \times 3 \times 13$	$82 = 2 \times 41$	$88 = 2 \times 2 \times 2 \times 11$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$		

We can reject the numbers 1 bigger than 40, 46, 52, 58, 60, 72, 82, 88, 96 which leaves these possibilities:

$$43, \ 67, \ 71, \ 79.$$

We now consider the numbers 44, 68, 71 and 80 to see which is the product of a square and a cube:

$$44 = 4 \times 11 \qquad 68 = 2 \times 2 \times 17 \qquad 72 = 8 \times 9 \qquad 80 = 16 \times 5.$$

Only one of these has the required property, hence the age of the ex-teacher is 71.

Solution 2
First we list the products of three distinct prime numbers starting with smallest:
$2 \times 3 \times 5 = 30$ (too small so reject)

(a) $2 \times 3 \times 7 = 42$ \qquad (b) $2 \times 3 \times 11 = 66$ \qquad (c) $2 \times 3 \times 13 = 78$

$2 \times 3 \times 17 = 102$ (too large so reject)

(d) $2 \times 5 \times 7 = 70$ \qquad $2 \times 5 - 11 = 110$ (too large so reject)

Now consider the numbers which are 1 more than each (a). (b), (c), (d). These give 43, 67, 71 and 79, which are all prime
We now consider the numbers 44, 68, 72 and 80 (2 more than each (a). (b), (c), (d)) to see which is the product of a square and a cube:

$$44 = 4 \times 11 \qquad 68 = 2 \times 2 \times 17 \qquad 72 = 8 \times 9 \qquad 80 = 16 \times 5.$$

Only one of these has the required property, hence the age of the ex-teacher is 71.

P2.2. Two adults, Uncle Jim and Mother, and two children, Nancy and Peggy, want to sail to Wild Cat Island. The boat is small and can only hold two individuals if they are **both** children.

How can all four of them get to the island?

Solution
Nancy and Peggy sail across, and Nancy sails back alone.
Then Uncle Jim sails across, and Peggy sails back alone.
Nancy and Peggy sail across, and Nancy sails back alone.
Then Mother sails across, and Peggy sails back alone.
Finally Nancy and Peggy cross again, and all four are on Wild Cat Island.

P2.3. John has a rectangular patio in his back garden. He increases both the length and the width of his patio by 20%.
(a) What is the percentage increase in the perimeter of the patio?
(b) What is the percentage increase in the area of the patio?

Solution
(a) Each edge is increased by 20% so their total is increased by 20%.

(b) After the length is increased it will be 1.2 times it's original value.
The same is true for the width.
As $1.2 \times 1.2 = 1.44$, the area has been increased by 44%.