Primary Division 2016-2017 Round 3 Solutions

P3.1. Alice put some 10p coins on the table. Half of them were tails up. Alice turned over two of the coins, and then one third of them were tails up.

How many coins did Alice put on the table?

Solution

If Alice turned over two heads the number of tails would increase.

If Alice turned over a head and a tail the number of tails would stay the same.

So she must have turned over two tails. The proportion of tails decreases from $\frac{1}{2}$ to $\frac{1}{3}$, i.e. by $\frac{1}{6}$.

So $\frac{1}{6}$ of the coins were turned over.

So Alice put $6 \times 2 = 12$ coins on the table.

Guidelines

1 mark for a clear solution

1 mark for deciding she turned over 2 tails (to get 2 more heads)

1 mark for getting that $\frac{1}{6}$ were turned over.

1 mark for finishing.

P3.2. Colin and Tom are on a camping holiday and, at their campsite, they make friends with Fiona. They ask her when her birthday is but, being a bit of a joker, Fiona tells them only that it is one of the following dates.

May 14, May 15, May 18, June 16, June 19, July 12, July 15, August 12, August 14, August 16.

She then tells Colin the month of her birthday, but not the day in the month, whilst she tells Tom the day in the month, but not the month.

Immediately, Colin declares "Well, Tom certainly cannot know for sure when Fiona's birthday is" to which Tom replies "Ah, but now I do." "And now I know when it is as well," comes back Colin.

When is Fiona's birthday? Explain your reasoning.

Solution

If Fiona gave Tom day 18 or 19, then he would know immediately the exact date of the birthday (May 18 or June 19) as these are the only occurrences of 18 and 19. Since Colin is certain that Tom cannot be sure of the exact date, Tom concludes that Fiona must have given the month July or August to Colin.

As Tom now knows the full date exactly, Colin concludes that this must be one of July 15, August 14 or August 16 (it cannot be July 12 or August 12 as, knowing only the day in the month, Tom wouldn't be sure which of these it would be). Since Colin knows the month and says that he now knows for certain the exact date, this must be July 15 as, if Fiona had told him August, then both the dates August 14 and August 16 would still be possible.

Thus Fiona's birthday is July 15.

P3.3. The pages of George's book are numbered from 1. The page numbers have a total of 555 digits. How many pages does the book have?

Solution

Pages 1-9 contain 9 digits.

Pages 10-99 contain $2 \times 90 = 180$ digits

This leaves 555 - 189 = 366 digits at 3 digits per page.

So there are a further $366 \div 3 = 122$ pages.

So the book has 99 + 122 = 221 pages.