P2.1. Four coloured beads, red, yellow, green and blue, are attached to the corners of a square. Two arrangements like those shown are considered to be the same because one square can be placed on top of the other by rotating and/or flipping so that the colours match. What is the total number of different arrangements (including the
 one shown)?

## Solution

Opposite the red bead we can place either the yellow, green or blue bead. Whichever way round the remaining two beads are placed gives the same arrangement - by flipping about the diagonal. So there are 3 different arrangements.

P2.2. A shopkeeper has a set of weights with values 200 grams, 300 grams and 500 grams.

Using these weights he can weigh different amounts: for example he can weigh 400 grams by putting the 200 gram and 500 gram weights on one side of the scales and the 300 gram weight and the bag to be filled on the other side.


And of course he can put one or more of the weights on one side of the scales and a bag to be filled on the other side.
Show that using these weights he can weigh all amounts from 100 grams to 1000 grams in steps of 100 grams except for one.

## Solution

Omitting the 100 s and letting the filled bag weight be $x$ hundred grams, the table shows how to obtain 100 g up to an including 800 g and also 1000 g .

| $x$ | weights in the left pan | weights in the right pan |
| :--- | :--- | :--- |
| 1 | 2 | 1 and $x$ |
| 2 | 2 | $x$ |
| 3 | 3 | $x$ |
| 4 | 2 and 5 | 3 and $x$ |
| 5 | 5 | $x$ |
| 6 | 3 and 5 | 2 and $x$ |
| 7 | 2 and 5 | $x$ |
| 8 | 3 and 5 | $x$ |
| 9 | $?$ |  |
| 10 | 2,3 and 5 |  |

So the amount that cannot be weighed is 900 grams.
$\mathbf{P 2 . 3}$. James thought of three numbers. He told his friend that the first and second numbers added up to 37 , the second and third added up to 53 , and the first and third added up to 78 . What were the numbers which James thought of?

## Solution

If we add 37, 53 and 78, we will have counted each of James's numbers twice.

$$
37+53+78=168
$$

so the total of the original numbers is 84 .

The first and second numbers add up to 37 , so the third number is $84-37=47$.

Hence the first number is $78-47=31$ and the second number is $53-47=6$.

