Primary Division 2016-2017 Round 1 Solutions

P1.1. A school has fewer than 200 pupils.
When they line up in rows of 4 there is 1 extra pupil.
When they line up in rows of 5 there are 2 extra pupils.
When they line up in rows of 6 there are 3 extra pupils.

How many pupils could there be in the school?

Solution
Rows of 4: 5, 9, 13, 17, …
Rows of 5: 17 pupils will leave 2 over
Rows of 6: for there to be 3 people left over the number must be divisible by 3 and be odd.
So 17 works for rows of 4 and 5 but not 6.
But if we add $5 \times 4 (= 20)$, it will still work for rows of 4 and 5.
So we get 37, 57, … and 57 is divisible by 3.
Thus 57 is one possible answer.
{Other possibilities are obtained by adding the lowest common multiple of 4, 5 and 6, i.e. 60.
So the possible numbers of members are 57, 117 and 177.}

Alternative solution:
(i) Rows of 5: Multiples of 5 all end in 5 or 0 so the situation with rows of 5 with 2 left over gives a total ending in 7 or 2.
(ii) Rows of 6: for there to be 3 left over the total must be odd and a multiple of 3.
So from (i) the total must and in 7: 7, 17, 27, 37, 47, 57,....., 187, 197.

Check for those that are multiples of 3: 27, 57, 87, 117, 147, 177.

(iii) Rows of 4: for there to be 1 person left over the total has to be 57, 117 or 177.

P1.2. Professor A. M. Nesia has a safe with a combination lock. In her journal, the note she uses to help her remember is this diagram $\rightarrow$
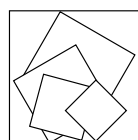
and the year of her birth, 1941,
This reminds her that the code is a sequence of five perfect squares (square numbers) in ascending order where the mean = 19, median = 4 and mode = 1.
Find the combination.
Solution
With five numbers written in numerical order, the third one must be the median: _, _, 4, _, _
The mode is 1 and this has to be before the 4 so both numbers less than 4 must be 1: 1, 1, 4, _, _

Let the last two values be a and b.
The mean is 19 so the total $= 1 + 1 + 4 + a + b = 19 \times 5$, i.e. $a + b = 89$.

We need two perfect squares which add up to 89:

Square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81 and to get a last digit of 9 it has to be 4 and 5 ie $a = 25$ and $b = 64$.

So the combination is 1142564.

P1.3. Fiona and Peter enjoy playing the game Snakes and Ladders.

In one game they noticed that after they had both thrown twice they were both at the foot of the same ladder which took them to square 53.

After two more throws each, they both arrived at the head of a snake and moved down to square 4.

Each time, they threw completely different numbers. The total of the first two throws was 4 more than the total of the second two throws. Also, in the four throws, Fiona did not roll the same number more than once and neither did Peter.

Which numbered square was at the foot of the ladder and which square was at the top of the snake?
Solution

Totals for different numbers in 2 throws are shown in the table:

Lower die		1	2	3	4	5
Higher die	**1**					
	2	3				
	3	4	5			
	4	5	6	7		
	5	6	7	8	9	
	6	7	8	9	10	11

Totals differ by 4 so 4,5 (or 5,4) and 6,3 (or 3,6) paired with 1,4 (or 4,1) and 2,3 (or 3,2).

First two throws total 9 so foot of ladder is on square 9. Now on square 53 and total of next two throws is 5 so head of snake is on square 58.