2015-2016 Primary Solutions Round II

P3.1. Maureen, Alice and Siobhan are three young sisters, in that order of age. Alice is two years older than Siobhan.

Each year, their wealthy aunt gives each of them, for each year of her age, as many pounds as she is years old. For example, on her first birthday a girl would receive one pound and on her third birthday nine pounds. The aunt has promised to continue this family custom with each girl until her twelfth birthday.

This year Maureen received as much as Alice and Siobhan put together.

How much will Siobhan receive next year?

Solution

The amounts received creates a list of the square numbers:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121 and 144.

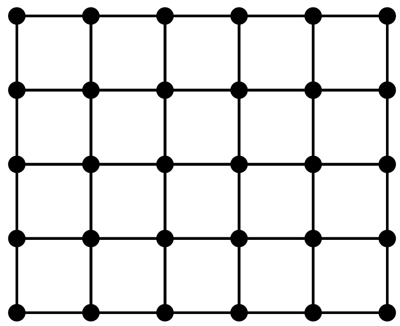
Siobhan	Alice	total	
1	9	10	
4	16	20	
9	25	34	
16	36	52	
25	49	74	
36	64	100	YES
49	81	130	
64	100	164	
81	121	202	
100	144	244	

So we are looking for total = possible amount for Maureen,
i.e. a perfect square up to 144

The only possible solution is 6, 8 and 10.

Siobhan will receive £49 next year.

P3.2.



The diagram represents a rectangular net. The net is made from string knotted together at the points shown. The strings are cut a number of times; each cut severs precisely one section of string between two adjacent knots. What is the largest number of such cuts that can be made without splitting the net into two separate pieces?

Solution

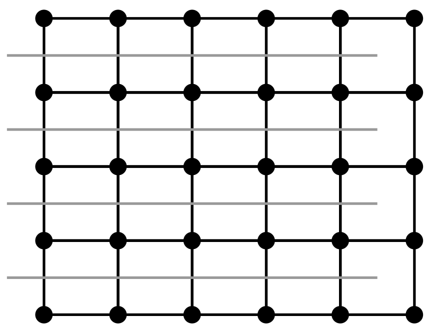
There are $6 \times 5 = 30$ knots.

To connect these 30 knots requires a minimum of 29 strings.

There are $5 \times 5 = 25$ horizontal strings, and $6 \times 4 = 24$ vertical strings, 49 strings in all.

So at most $49 - 29 = 20$ strings can be cut.

This is possible:



P3.3. In a diving competition, five judges each award a whole-number score from 1 to 10 and an average mark is then calculated. However there are three different ways of measuring the average: mean, mode and median. After a particular set of scores were given, an argument arose as to which measure should be used, as this would lead to three different final marks being awarded: 7, 8 or 9. Work out all the different possible scores that could have been awarded. Which mark would match with each measure?

Solutions

If the **mean** is 9, the scores could be 9, 9, 9, 9, 9 or 8, 9, 9, 9, 10 or 8, 8, 9, 10, 10 or 7, 9, 9, 10, 10. But then the median would also be 9, whereas it must be 7 or 8.
So the mean is **not** 9.

If the **median** is 9, the three highest scores must be
9, 9, 9 making the mode 9 also
or 9, 9, 10 making the mode 9 also (or undefined)
or 9, 10, 10 making the mode 10 (or undefined).
So the median is **not** 9.

Hence the **mode** must be 9.

If there are three or more 9s the median must be 9 also.
If the highest numbers are 9, 9, 10 the median must be 9 also.
So the highest numbers must be two 9s with no other repeats.

If the median is 8 then the three highest numbers are 8, 9, 9.

To make the mean 7, the other two numbers must add to $7 \times 5 - 8 - 9 - 9 = 9$.
So the possibilities are 4, 5 or 3, 6 or 2, 7 (but not 1, 8 as the mode would be undefined).

If the median is 7, then the three highest numbers are 7, 9, 9.
To make the mean 8 the other two numbers must add to $8 \times 5 - 7 - 9 - 9 = 15$.
But no two numbers less than or equal to 7 add to 15.

So there are exactly three valid score combinations which are:

	Mean	Median	Mode
4, 5, 8, 9, 9	7	8	9
3, 6, 8, 9, 9	7	8	9
2, 7, 8, 9, 9	7	8	9

In each case the mean is 7; the median is 8; and, the mode is 9.