2014-2015 Primary Solutions Round III

P3.1

Dancers in a class are spaced evenly around a circle and are then counted off consecutively from number 1. Dancer 20 is directly opposite dancer 53. How many dancers are there in the group?

Solution

Dancer 20 is directly opposite dancer 53, and so there are $53 - 20 = 33$ dancers in half the circle, and 66 in all.

P3.2

In a singles tennis tournament there are 10 players. The organiser needs to arrange the 10 players into 5 pairs for the first round. In how many ways can this first round be drawn up?

Solution

Select one player. Then there are 9 possible choices for his opponent.

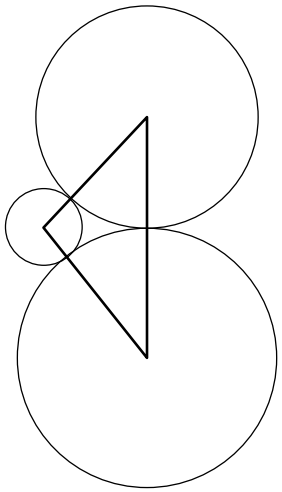
Now select any one of the remaining players. There are 7 choices left for his opponent.

… until the last two players form the last pair.

So the number of ways is $9 \times 7 \times 5 \times 3 \times 1 = 945$.

P3.3

In the diagram (which is not drawn to scale) the lengths of the sides of the triangle are 8, 9 and 13 centimetres. The centres of the circles are at the vertices of the triangle, and the circles just touch. Find the radius of the largest circle.



Solution

Let the radii of the large, medium and small circles be l, m and s centimetres respectively.

Then

$$l + m = 13$$

$$l + s = 9$$

$$m + s = 8$$

Adding the first two equations:

$$2l + m + s = 22$$

$$2l = 22 - (m + s) = 22 - 8 = 14$$

$$l = 7$$

i.e. the largest circle has radius 7 cm.

Alternative Solution

Let the radii of the large, medium and small circles be l, m and s centimetres respectively.

Then

$$l + m = 13$$

$$l + s = 9$$

$$m + s = 8$$

Adding these gives

$$2l + 2m + 2s = 13 + 9 + 8 = 30$$

$$\text{so} \quad l + m + s = 15$$

$$\text{using } l + m = 13 \text{ gives } s = 2$$

$$\text{using } l + s = 9 \text{ gives } m = 6$$

$$\text{using } m + s = 8 \text{ gives } l = 7$$

i.e. the largest circle has radius 7 cm.