2012-13 Primary Set 3 solutions

P3.1.

To encourage Lazy Leonard to work at a job for 36 days, it was decided that he would be paid £4 per day for each day he worked, but he would forfeit £5 for every day he idled. At the end of the 36 days his pay was £0.

How many days did he work and how many days did he idle?

Solution 1

Consider a possible sequence of work days (W) and lazy days (I).

W W W W W I I I I

At the end of these 9 days, Leonard worked have gained 5 × £4 and lost 4 × £5. Which is £0. The order of work and lazy days does not matter so long as there are 5 and 4.

Since 36 days is four times as long, to 'break even' Leonard must have worked on $4 \times 5 (= 20)$ days and been idle on $4 \times 4 (= 16)$ days.

Solution 2

If Leonard works for x days, then he is idle for $36 - x$ days.

Money earned = £$4x$.

Money lost = £$5(36 - x)$.

$$4x = 5(36 - x)$$

$$4x = 180 - 5x$$

$$9x = 180$$

$$x = 20$$

So Len worked for 20 days and was idle for 16 days.

P3.2.

Andrew has forgotten the code to allow him to unlock his bicycle.
He knows:

- it is a 3-digit number;
- the sum of the digits is 13;
- the outer digits are even;
- subject to these conditions, the product of the digits is as large as possible.

What is the code? Explain clearly.

Solution

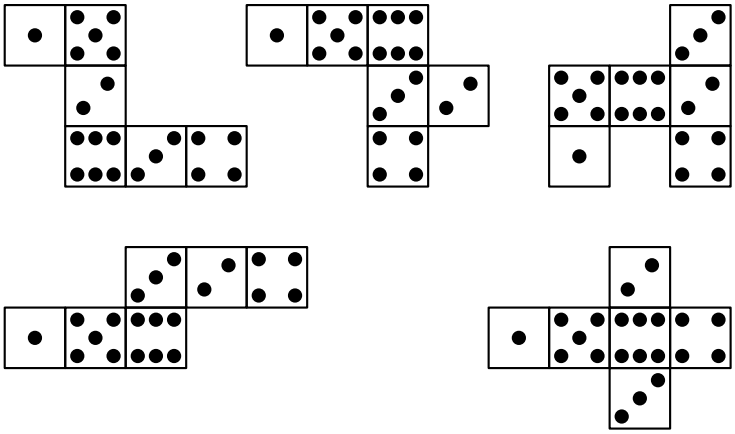
The middle digit is odd so it can only be 1, 3, 5, 7 or 9.
The first digit is 2, 4, 6 or 8.
The table shows possible codes with an odd middle digit and a digit sum of 13. The product is shown in brackets

2 1 impossible	2 3 8 (48)	2 5 6 (60)	2 7 4 (56)	2 9 2 (36)
4 1 8 (32)	4 3 6 (72)	4 5 4 (80)	4 7 2 (56)	4 9 impossible
6 1 6 (36)	6 3 4 (72)	6 5 2 (60)	6 7 impossible	6 9 impossible
8 1 4 (32)	8 3 2 (48)	8 5 impossible	8 7 impossible	8 9 impossible

The biggest product is 80 which is the outcome from the triple 4 5 4.
So the code is 4 5 4.

P3.3.

A cube can be opened out into a net made up of six squares. But which nets below made up of six squares fold up into a cube? For those that do not, use the spots to explain why not. For those that do, with the given spots, which form a standard dice? Explain.



Solution

Numbering the top 3 the nets 1 to 3 and the others 4 and 5, we have:

1. Not a cube as faces with 4 and 5 dots overlap.
2. Not a cube as faces with 1 and 4 dots overlap.
3. Not a cube as faces with 1 and 4 dots overlap.
4. A cube and a dice as faces with 1 and 6, 2 and 5, 3 and 4 dots are opposites.
5. A cube but not a dice as faces with 2 and 3 dots are opposites (so are 4 and 5).

The last two nets will make cubes but only the first net in the bottom row will be a dice.