2011-2012 Primary Set 2 solutions

P2.1. The teacher has set a problem about what weights you can measure with five weights: 1, 2 , 3 , 4 and 5 g but you decide this is too easy!

If you have five weights, what would be the maximum weight you can weigh any weight up to and including it: what would the five weights have to be ?

Solution

To weigh an object which weighs 1 g, you need that weight.

To weigh an object which weighs 2 g, you need either another 1 g weight or a 2 g weight.

To weigh an object which weighs 3 g, you can do it with 1 g and 2 g weights.

To weigh an object which weighs 4 g, you need a 4 g weight.

With 1, 2 and 4 g weights you can weigh $5 (= 1 + 4)$, $6 (= 2 + 4)$ and $7 (= 1 + 2 + 4)$.

The next weight will have to be 8 g and this permits:

9	10	11	12	13	14	15
8 + 1	8 + 2	8 + 2 + 1	8 + 4	8 + 4 + 1	8 + 4 + 2	8 + 4 + 2 + 1

So the fifth weight should be 16 g. By combining it with each of the lesser weights in turn we can see that the five weights 1, 2, 4, 8 and 16 can weigh anything from 1 to 31g.

P2.2. At noon, the hour and minute hands of a clock point in the same direction. Determine, to the nearest second, the last time before noon that the hour and minute hands point in the same direction.

Also determine, to the nearest second, the last time before noon that the hour and minute hands point in exactly opposite directions.

Explain your reasoning.

Solution

In 12 hours the minute hand makes 12 revolutions and the hour hand makes one revolution. The hands point in the same direction 11 times during this period, equally spaced round the clock face. Thus the times are 0 hours, 1/11(12 hours), 2/11(12 hours), … , 10/11(12 hours).

So the last time before noon is after

10/11(12 hours)
= 10 10/11 hours
= 10 hours and 10/11 (60 minutes)
= 10 hours and 54 6/11 minutes
= 10 hours, 54 minutes and 6/11 (60 seconds)
= 10 hours, 54 minutes and 33 seconds

So the time is 10.54 and 33 seconds.

The hands point in opposite directions at 6 o'clock, and will point in opposite directions again 11 times before reaching 6 o'clock again. The time just before noon is after

6 hours + 5/11 (12 hours)
= 11 5/11 hours
= 11 hours and 27 3/11 minutes
= 11 hours, 27 minutes and 16 seconds

So the time is 11.27 and 16 seconds.

P2.3. A giant timeline is constructed, showing every year from 1AD until 2012. The company which has to make this needs to calculate how many of each digit 0 – 9 will be required. Assuming that no leading zeroes are necessary, how many of each digit will be needed?

Explain your reasoning.

Solution

Consider the years 000 – 999 (using leading zeroes). There are 1000 numbers each with 3 digts so there are 3000 digits in total, 300 of each. But we have to allow for 111 leading zeros.

Now taking 1000 – 1999, another 300 of each digit required, excepting 1, which will be needed 1300 times.

Finally, 2000 - 2012

0 -	25
1 -	5
2 -	15
3 to 9	1

Overall,

	0	1	2	3	4	5	6	7	8	9
0 to 999	189	300	300	300	300	300	300	300	300	300
1000 to 1999	300	1300	300	300	300	300	300	300	300	300
2000 to 2012	25	5	15	1	1	1	1	1	1	1
	514	1605	615	601	601	601	601	601	601	601