Primary Division: Problems III

P3.1. I have some coins in my pocket. Using the three clues below, work out how many coins I have.

- If the number of coins I have is not a multiple of 10, then it is a number between 30 and 39.
- If the number of coins I have is a multiple of 5, it is a number between 1 and 19
- If the number of coins I have is not a multiple of 8, then it is a number between 20 and 29

You must a give a full explanation of your solution.

Solution

From the first clue, the number of coins is   10, 20, 30, ... or a number between 30 and 39.
From the second clue, if the number of coins is a multiple of 5, that is 5, 10, 15, ... then it must lie between 1 and 19.
So the only numbers which are consistent with the first two clues are

$$10, 31, 32, 33, 34, 36, 37, 38, 39$$

From the third clue, the number is either a multiple of 8 or it lies between 20 and 29; no valid number is between 20 and 29 so the number of coins must be a multiple of 8, that is the number is 32 being the only multiples of 8 in the list above.

P3.2. In a game of three-dimensional noughts and crosses how many possible winning lines can there be?

Solution

A winning line consists of three in a row where row can be horizontal, vertical or sloping.
There are 9 vertical lines, 9 horizontal lines from left to right and 9 horizontal lines from back to front.
On the front face, there are 2 diagonal lines and each of these has 2 more which are parallel to it, giving 6. In a similar way, there are 2 more sets of 6.
Finally, there are the 4 main diagonals.
Total =   27 + 18 + 4 = 49.

P3.3. In Wuppertal Zoo, there is a tank containing 41 spectacular tiger fish. Each male fish has 111 stripes while each female has only 37 stripes. Unfortunately the male fish caught a disease and two thirds of them died. How many stripes were on display in the tank after this?

Solution

Note that 111 is three times 37 so each male fish had three times as many stripes as a female fish. When two thirds of the male fish die, two thirds of the male stripes disappear. This gives the same number of stripes on male fish as we would have if **all the male fish survived** but had lost two thirds of their stripes and so had only 37 stripes instead of 111.
So the number of stripes now on display is $41 \times 37 = 1517$.