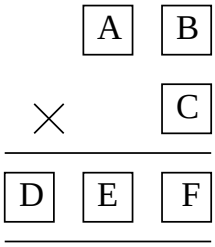
2009 Primary Set 2 solutions

P2.1. The digits 1, 2, 3, 4, 5 and 6 can be placed in the spaces shown below to create a correct calculation.
Can you find it?

Explain how you reached your answer.



Solution

Rewrite the calculation as $AB \times C = DEF$, where A, B, C, D, E and F are the digits 1, 2, 3, 4, 5 and 6 in some order.

Consider the units column.

Neither B nor C can be 1 as then F would be a copy of the other.

Neither B nor C can be 5 since F would be 5 or 0.

Neither B nor C can be 6 since multiplying by 2, 3 and 4 would produce 2 or 8 or 4 for F.

Taking possible pairs for B × C gives:

(a) $2 \times 3 = 6$ (b) $2 \times 4 = 8$ (c) $3 \times 4 = 12$

(b) is impossible and (a) and (c) mean that 2 and 3 are used up.

Considering D, A could be 6 and C could be 4 but, even if they were, D cannot exceed 2. But we know 2 is in the units column so D = 1.

	A	2, 3 or 4
		2, 3 or 4
1	E	2 or 6

So one of A and E are 5.

If E = 5 then the bottom number is 152 or 156. Dividing these by the possible values of C involves numbers other than 1, 2, 3, 4, 5, 6.

So it is A that is 5 and the top number is 52 or 53 or 54. So we have

AB	52	53	53	54
C	3	2	4	3
DEF	156	106	208	162

Thus the only solution is : $54 \times 3 = 162$.

P2.2. Example:

The (2 + 3)th	Square number is	25
	Triangular number is	15
	Cube number is	125

Using each of the digits 1 to 9 exactly once, fill the boxes below to obtain a true statement

The (☐ + ☐)th	Square number is	☐☐
	Triangular number is	☐☐
	Cube number is	☐☐☐

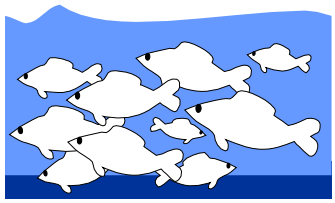
Solution

Number	4th	5th	6th	7th	8th	9th	10th
Square numbers	16	25	36	49	64	81	
Cube numbers		125	216	343	512	729	
Triangular numbers	10	15	21	28	36	45	55
Unused digits						3, 6	

The column with no duplicated digits is the 9th
So the answer is

The (3 + 6)th	Square number is	81
	Triangular number is	45
	Cube number is	729

P2.3. Under the sea in the land of Swimmington there was a school of mermaids, octopuses and seahorses. Assuming that the tentacles on an octopus can also be called arms, the group had 16 tails, 50 arms and 40 eyes. How many of each are in the group?



Solution

Altogether there are 40 eyes so there are 20 in the school.
Octopuses don't have tails so there are 16 mermaids and seahorses and so 4 octopuses.
Seahorses don't have arms and 4 octopuses will have 32 arms showing there are 18 arms for the mermaids.
So there are 9 mermaids and therefore 7 seahorses.

Alternative
Let the numbers of octopuses, mermaids and seahorses be O, M and S respectively.
40 eyes mean 20 creatures so:

$$O + M + S = 20 \qquad (1)$$

16 tails mean

$$M + S = 16$$

Thus

$$O = 4$$

Considering the 'arms'

$$8O + 2M = 50$$

$$32 + 2M = 50$$

$$2M = 18$$

$$M = 9$$

Using (1)

$$4 + 9 + S = 20$$

so

$$S = 7$$

So there are 9 mermaids, 4 octopuses and 7 seahorses.