P1.1 From his home $H$, a travelling salesman has to make 12 deliveries, 4 to each of the towns $A, B$ and $C$ as shown on the diagram.


For his travel he gets a mileage allowance of 50p per mile. He claims for 4 return journeys from home to each of the towns $A, B$ and $C$. But he actually made the deliveries on a total of 4 trips, each trip going from $H$ to $A$ to $B$ to $A$ to $C$ to $A$ and back to $H$. If, by his dishonest claim, he made an additional $£ 160$, how far is it from $H$ to $A$ ?

## Solution

In his actual 4 trips, he travelled the road from A to B and back 4 times, the road from A to C and back 4 times and the road from H to A and back 4 times.
In the 4 journeys to each of $\mathrm{A}, \mathrm{B}$ and C that he claimed for, he travelled the road from A to B and back 4 times, the road from A to C and back 4 times and the road from H to A and back 12 times. So he claimed for extra mileage equal to 8 return journeys from H to A which is 16 times the distance from H to A. Since he got an additional $£ 160$, the distance from H to A earned him $£ 10$. At 50 p per mile that distance must be 20 miles.

P1.2 Four old-timers, John, Willie, Andy and Greig, take their wives to a tea-dance. At one point during the dance, the partners were as follows:

Flo is dancing with Willie.
Hettie is dancing with Beeb's husband
Mary is dancing with Hettie's husband.
Andy is dancing with Greig's wife.
Greig is dancing with Willie's wife.
Identify the married couples.

## Solution

Since Flo is dancing with Willie, by the next two statements, Willie cannot be Beeb's husband (as Hettie is dancing with him) and also cannot be Hettie's husband (as Mary is dancing with him). Willie cannot be married to Flo, since by the last statement Greig is dancing with Willie's wife. So Willie has to be married to Mary.
As Greig is dancing with Mary, by the third statement, Greig is Hettie's husband. So Andy is dancing with Hettie and by the second statement Andy is married to Beeb. That leaves John to be married to Flo. Thus:

| Willie | and | Mary | are married |
| :--- | :--- | :--- | :--- |
| Greig | and | Hettie | are married |
| Andy | and | Beeb | are married |
| John | and | Flo | are married |

P1.3 Two knights, Sir Anthony and Sir Stephen, each have a very small army. The total number in the two armies is fifty men. The number of mounted troops in Sir Stephen's army equals the number of foot soldiers in Sir Anthony's army. Sir Anthony has two fewer troops in total than Sir Stephen and the number of mounted troops in Sir Anthony's army is four fewer than the number of mounted troops in Sir Stephen's army. How many mounted troops and foot soldiers does each army have?

## Solution

The total in the two armies is 50 and there are 2 fewer in Sir Anthony's so the total in Sir Anthony's is 24 and in Sir Stephen's is 26.

Sir Anthony has 4 fewer mounted troops than Sir Stephen.
But Sir Anthony has the same number mounted troops as Sir Stephen has foot soldiers.
So Sir Anthony has 4 fewer mounted troops than foot soldiers.
His total is 24 , so he has 10 mounted troops and 14 foot soldiers.
Sir Stephen has 14 mounted troops and therefore he has $26-14=12$ foot soldiers.

## Alternative:

The total in the two armies is 50 and there are 2 fewer in Sir Anthony's so the total in Sir Anthony's is 24 and in Sir Stephen's is 26. Let the numbers in each category be:

|  | Sir Anthony's | Sir Stephen's |
| :--- | :--- | :--- |
| Foot soldiers | $a$ | $s$ |
| Mounted troops | $b$ | $t$ |

Now $a=t$ and $b+4=t=a=24-b$
so $b=10, t=14=a$ and $s=12$, i.e.

|  | Sir Anthony's | Sir Stephen's |
| :--- | :--- | :--- |
| Foot soldiers | 14 | 12 |
| Mounted troops | 14 | 10 |

