2007-08 Primary Set 3 solutions

P3.1. Mary has three sons. All three are less than 10 years in age and the product of the ages of the younger two equals the age of the oldest. The sum of the ages of the three boys is a prime number. How old are the boys?
Explain your reasoning.

Solution

All are the boys are under 10 and the product of the two younger ages gives the age of the oldest.

We are not told that the younger two have different ages so the possible ages are:

2, 2, 4 or 2, 3, 6 or 2, 4, 8 or 3, 3, 9 and

these have totals: 8, 11, 14, 15

and 11 is the only prime number.

So the ages are: 2, 3 and 6.

P3.2. Oor Wullie and his pals are exploring in the jungle and have to cross a rope bridge at midnight. Unfortunately the bridge is only strong enough to support two people at a time. As it is dark, they also need a torch to be used every time the bridge is crossed but they only have one torch. Wullie can cross the bridge in five minutes, Wee Eck can cross in seven minutes and Fat Bob can cross in eleven minutes. But it takes PC Murdoch twenty minutes to get across. How quickly can all four get across the bridge?
Explain your reasoning.

Solution

We need to note that whenever two boys cross together, the time taken is that of the slower one. There are two equal, shortest times:

(a)	Wullie and Wee Eck cross together	7 minutes
	Wullie returns with the torch	5 minutes
	Fat Bob and PC Murdoch cross together	20 minutes
	Wee Eck returns with the torch	7 minutes
	Wullie and Wee Eck cross back	7 minutes
		46 minutes

Or, interchanging Wullie and Wee Eck returning with the torch.

(b)	Wullie and Wee Eck cross together	7 minutes
	Wee Eck returns with the torch	7 minutes
	Fat Bob and PC Murdoch cross together	20 minutes
	Wullie returns with the torch	5 minutes
	Wullie and Wee Eck cross back	7 minutes
		46 minutes

If Fat Bob and PC Murdoch do not cross together, for a shortest trip, Wullie should accompany them. Then Wee Eck must also cross with Wullie and that would be a total of 11 plus 20 plus 7 plus twice 5 making 48.

P3.3. A palindromic number is a number which reads the same backwards and forwards, for example 838 and 14541. As generally we do not write numbers with an initial zero, numbers such as 070 will not be included here.

(a) Which are there more of: 10-digit or 11-digit palindromic numbers?

(b) Which are there more of: 11-digit or 12-digit palindromic numbers?

Explain your reasoning.

Solution

(a) If you start with an 10-digit palindromic number, you can construct an 11-digit palindromic number by putting any digit between the 5th and 6th digits. It is possible to use any of the 10 digits 0, 1, . . , 8, 9 so **there are 10 times as many 11-digit than 10-digit palindromes**.

(b) All 12-digit palindromic numbers have a double digit in the middle and you can only get an 11-digit palindromic number by removing one of these so there are the same number of 11 and 12-digit palindromes.

OR

To construct a 12-digit palindromic number from an 11-digit palindromic number, you need to put a copy of the middle digit immediately after the middle digit. This will give a 12-digit palindromic number but ensures that you have the same number of each.

Another Solution

For a 10-digit palindromic number the first digit must be 1 to 9 and the next four can be 0 to 9.

The rest are then automatically determined. So the number of 10-digit palindromes is $9 \times 10 \times 10 \times 10 \times 10 = 9 \times 10^4$.

For 11 and 12-digit palindromic numbers the first digit must be 1 to 9 and the next five can be 0 to 9. The rest are then automatically determined. So the number of 11-digit palindromes is $9 \times 10 \times 10 \times 10 \times 10 \times 10 = 9 \times 10^5$.

Hence there are more 11-digit palindromic numbers than 10-digit palindromic numbers but equal numbers of 11 and 12-digit palindromic numbers.