2007 Primary Set 2 solutions

P2.1. At the Post Office in the village of Prime it is only possible to buy 5p and 7p stamps. If just these stamps are available, it is clearly impossible to get all possible values of postage. For example, you are unable to buy stamps to the value of 1p, 2p, 3p, 4p, 6p, 8p or 9p. But what is the **largest** amount of postage you are **unable** to obtain in the stamps available?
Explain your answer.

Solution

Start by listing possible values.

1p	No	2p	No	3p	No	4p	No
5p	Yes	6p	No	7p	Yes	8p	No
9p	No	10p	2×5	11p	No	12p	5+7
13p	No	14p	2×7	15p	3×5	16p	No
17p	2×5+7	18p	No	19p	5+2×7	20p	4×5p
21p	3×7	22p	3×5+7	23p	No	24p	2×5+2×7
25p	5×5	26p	5+3×7	27p	4×5+7	28p	4×7

There is no need to go further. The last five values 24p, 25p, 26p, 27p and 28p are all obtainable. Adding a 5p stamp to each of these will give 29p, 30p, 31p, 32p and 33p and this process can be continued to obtain all higher values.
So the biggest value which cannot be obtained using 5p and 7p stamps is 23p.

P2.2. Mrs MacDonald has four sons. Sandy, who is the tallest, is older than Michael, who is the lightest. Calum is the oldest but is shorter than James who is the heaviest. No one has the same rank (first through to fourth) in more than one category. Rank the four boys in each category: Age, Height and Weight.

Solution

From the given information, we can immediately fill in

Rank	Height	Age	Weight
1	Sandy	Calum	James
2			
3			
4			Michael

Sandy is older than Michael and, since Michael cannot be the youngest (as he is the lightest), Michael is the third eldest and Sandy the second eldest. This means that James is the youngest and that Calum has to be the shortest.

Rank	Height	Age	Weight
1	Sandy	Calum	James
2		Sandy	
3		Michael	
4	Calum	James	Michael

As each boy has to appear in each column and can only appear once in each row, Sandy is the third heaviest. The rest then fill in automatically.

Rank	Height	Age	Weight
1	Sandy	Calum	James
2	Michael	Sandy	Calum
3	James	Michael	Sandy
4	Calum	James	Michael

P2.3. Mr Green and his son Michael live next door to each other. Each has a big garden and his own mower. Mr Green can mow his lawn with the little mower in 45 minutes. His son Michael can handle the big mower and takes 30 minutes to do the same job. Last Saturday they were in a rush so they worked together using both mowers. How long did it take for them to mow the lawn?

Solution

In 1 minute, Mr Green can mow $\frac{1}{45}$ of the lawn and Michael can mow $\frac{1}{30}$ of the lawn. So, working together, in 1 minute, they can mow $\left(\frac{1}{45} + \frac{1}{30}\right) = \frac{5}{90} = \frac{1}{18}$.

So working together they will take 18 minutes.