2007 Primary Set 1 solutions

P1.1. Jack and Kylie are playing a game of table tennis. If Jack loses this game, he will have won the same number of games as Kylie. If Jack wins the game, he will have won twice as many games as Kylie.

How many games had each won before they started this game?

Explain your reasoning.

Solution

Before the start of the game, the first condition tells us that Jack has won 1 more game than Kylie and the second condition tells us that Jack has won 1 less than twice the number that Kylie has won. So

$$\text{The number that Kylie has won} + 1 = 2 \times \text{The number that Kylie has won} - 1$$
$$1 = \text{The number that Kylie has won} - 1$$

so Kylie has won 2 games and therefore Jack has won 3 games.

Using algebra:

Suppose that Jack has won j games and Kylie has won k games.

Then $j = k + 1$ and $j + 1 = 2k$.

So $k = 2$ and $j = 3$.

So Kylie has won 2 games and therefore Jack has won 3 games..

P1.2. Three friends visit a museum and walk up a flight of stairs.

Ross goes up one step at a time starting with his left foot on the first step. Sheila goes up two steps at a time starting with her left foot on the second step and Tom starts with his left foot on the third step and goes up three steps at a time.

Investigate these questions and explain your answers.

(a) Which is the first step that all three will tread on?

(b) Which is the first step that all three will tread on with their right foot?

(c) Which is the first step that all three will tread on with their left foot?

Solution

Ross puts his left foot on steps: 1, 3, 5 . . . and his right foot on steps: 2, 4, 6, . . .

Sheila puts her left foot on steps: 2, 6, 10 . . . and her right foot on steps: 4, 8, 12, . . .

Tom puts his left foot on steps: 3, 9, 15, . . . and his right foot on steps: 6, 12, 18, . . .

(a) 6

(b) 12

(c) No solution as Ross steps on odd numbered steps with his left foot and Sheila only steps on even numbered steps with her left foot.

Step	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Ross	L	R	L	R	L	R	L	R	L	R	L	R
Sheila		L		R		L		R		L		R
Tom			L			R			L			R

P1.3. A group of boys found a chestnut tree with the chestnuts just ready for picking. One of the boys climbed the tree and was able to knock down some chestnuts. He had just enough to give himself and the other boys three chestnuts each, with none left over. Then three of their friends joined them. They found that it was not possible to share the chestnuts evenly among the group.

However, when one more chestnut was picked, it was possible to give each boy two chestnuts, with none left over. How many boys were there altogether? **Explain your reasoning.**

Solution

(a)	Number of boys at start	3	4	5	6	. . .
(b)	Number of chestnuts at start	9	12	15	18	
(c)	Original number of boys + 3	6	7	8	9	
(d)	Twice row (c)	12	14	16	18	

The table shows the condition is satisfied (i.e. row (d) is 1 more than row (b) are equal) when there are 5 boys at the start and 8 boys altogether.

Using algebra:

Suppose that there were n boys at the start so that there were $3n$ chestnuts. After the other boys arrived, there were $n + 3$ boys and $3n + 1$ chestnuts.

Hence, $3n + 1 = 2(n + 3)$ so $n = 5$.

Total number of boys = 8.