2006 Primary Set 3 solutions

P3.1. A quiz had only 3-point questions and 5-point questions. The best possible score is 100 and there are 26 questions. How many of each type are there?

Solution

Since 3 × Number of 3-point questions plus 5 × Number of 5-point questions is 100, the number of 3-point questions must be a multiple of 5.

We also note that there cannot be more than 20 5-point questions and so there must be at least 6 3-point questions. Hence there must be at least 10 3-point questions. This gives the following table:

No. of 3-point Qs :	10	15	20	25
No. of 5-point Qs :	16	11	6	1
Total score :	110	100	90	80

Thus there are 15 3-point questions and 11 5-point questions.

OR

If all 26 questions were worth 3 points, the total would be 78 points.

This is 22 points short but a 5-point question is worth 2 more than a 3-point question.

Thus, dividing 22 by 2 means that there have to be 11 5-point questions and hence 15 3-point questions.

P3.2. In cleaning out a drawer, Mrs Smith found two old watches which she and her husband had discarded. She wound them up, and, after setting them accurately, started both watches at the same time. An hour later she noticed that her old watch had gained one minute while her husband's had lost two minutes. Checking them from time to time, it was clear that her old watch was running consistently fast and her husband's consistently slow. Next morning, when she looked at the watches again, it was 7 o'clock on her old watch and 6 o'clock on her husband's. What time was it when she started the watches running?

Solution

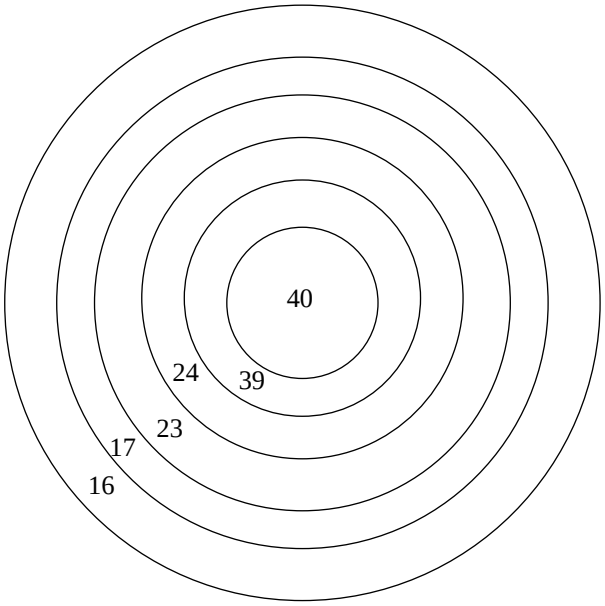
Every hour Mrs Smith's watch gained 1 minute and her husband's lost 2 minutes, so they differed by 3 minutes. So they would differ by an hour after a multiple of 20 hours.

Since it was the next day when she consulted the watches, that must have been after 20 hours. At that time her watch showed 7.00 a.m. but had gained 20 minutes. So the actual time was 6.40 a.m. So the watches were started 20 hours earlier at 10.40 a.m.

P3.3. On an archery target, the scoring is 40 for the bull's-eye and 39, 24, 23, 17 and 16 respectively for the rings from the centre outwards, as shown. Three players had a match with six arrows each. The result was as follows:

Wendy – 120 points
Pat – 110 points
Bill – 100 points

Every arrow scored, and the bull's-eye was only once hit. Determine the exact six hits made by each competitor.



Solution

Since each player score with every arrow, the only place for bull's-eye is in the 120 score because 5 of the lowest score (16) exceeds both $110 - 40$ and $100 - 40$.

So Wendy scored 40 and 5×16.

The same argument shows that 39 is not obtained by either Pat or Bill.

As each player scored with each arrow, we can consider ways of getting more than $6 \times 16 = 96$.

For Pat, 14 more points are needed, choosing 6 values from the set {0,1,7,8}. The only possible way is $7 + 7 + 0 + 0 + 0 + 0$ since 8 is 6 short etc.

So Pat scored 2×23 and 4×16.

For Bill, we need 4 points, choosing 6 values from the set {0,1,7,8}. The only possible way is $1 + 1 + 1 + 1 + 0 + 0$.

So Bill scored 4×17 and 2×16.